

Interlacing Mathematics and Culture: Symmetry in Traditional Pavements and Crafts

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Abstract

In this paper, the authors interlace the work they have been developing in the last years, namely the classification of the Portuguese pavement patterns in the Azores islands (Teixeira, 2015; Teixeira, Costa & Moniz, 2015), with the exploration of symmetries in patchwork and ceramics within a set of professional development courses for mathematics teachers held in Aveiro, in the north of mainland Portugal (Hall, 2016).

This paper focuses on rosette groups which in spite of being the simplest symmetry groups with only rotational and/or reflection symmetries, are rich enough to describe an endless variety of patterns/designs found in practice.

Keywords: symmetry; rosette groups; Portuguese pavement; patchwork; ceramics; ethnomathematics.

Symmetry: a principle of organization

The mathematical concept of symmetry is extremely fruitful in establishing links between mathematics and our daily life, since it provides a set of tools to interpret the reality surrounding us with a new look. It also provides an opportunity to analyze traits of our culture and to classify, in a rigorous way, the heritage left by our ancestors. Moreover, the mathematical classification of a figure, based on the identification of its symmetry group, can be a highly motivating activity, not only for any citizen but also for young people who find a way to apply the concepts they learn in school.

The intuitive idea of symmetry comes from an early age. When we look to our parents' faces or when we discover many surrounding objects, we are led to experience a sense of harmony and invariance (repetition) that holds our attention.

Over time, we begin to identify different types of symmetry, when we look in the mirror or when we see a reflected image in a lake (*reflection symmetry*), when we look at the sails of a windmill (*rotational symmetry*), when we pay attention to a motive that is repeated along a certain direction, in a quilt or a decoration frieze (*translational symmetry*), or when we see the set of footprints we leave behind us when walking barefoot in the sand (*glide reflection symmetry*).

As David Wade (2006) says, symmetry is completely entangled with asymmetry, just like order and disorder are; the author states that “symmetry principles are characterized by a quietude, a stillness that is somehow beyond the bustling world; yet, in one way or another, they are almost always involved with transformation, or disturbance, or movement” (p. 1). Symmetry has to do with repetition. Too much repetition is boring whereas too much change is stressful. A wise balance between the two is what makes life marvelous and exciting. We find symmetry everywhere we go and that is why a complete

understanding of symmetry was necessary and is now achieved, at least from a mathematical point of view.

Perfect symmetry cannot be found in reality, either because it requires infinity which is an abstract concept we do not find in practice, or because nature and human creations are intrinsically irregular or imperfect. Therefore, for the sake of simplicity, we will disregard minor flaws or irregularities when looking for symmetry in objects. The objects found in everyday life can be classified according to the symmetries they possess. We live in a 3-dimensional world but the study of symmetry in three dimensions is far more complex than it is in two. Many 3-dimensional objects can be seen in a 2-dimensional perspective and many of the patterns we are interested in are in fact 2-dimensional. Therefore, we will treat the examples analyzed in this paper as plane figures and restrict ourselves to symmetries in the plane.

Let us recall the mathematical/geometrical definition of symmetry. A *symmetry* of a figure is an isometry that leaves it invariant, that is, it is a geometric transformation (a point to point bijective application of the plane) that preserves distance and maps the figure onto itself (globally, not point by point).

The symmetries of a figure are therefore related to its invariance under certain transformations and it is this regularity which causes a pleasant feeling.

The set of symmetries of a figure F together with the operation composition forms a group which is known as the *symmetry group* of F . The identity element of the group is the identity (transformation), which maps each point onto itself. The identity can be seen as a trivial rotation of 0 degrees (or any multiple of 360 degrees).

Identifying the symmetry group of a figure allows a rigorous classification of that figure. A symmetry group can either be discrete or continuous. Most figures we are

interested in have discrete groups. The circle (or a set of concentric circles or annulus) or the straight line (or parallel lines) are exceptions whose sets of symmetries are continuous. In the plane there are only three categories of discrete symmetry groups: *rosette groups* (they have a finite number of symmetries which can only be rotations or reflexions); *frieze groups* (they have translation symmetry in only one direction; one can identify a motive which is replicated at a constant distance along a straight line); and *wallpaper pattern groups* (they have translation symmetry in two directions and cover the plane).

In this paper we shall only consider rosettes. Rosettes can be only of two types: either their symmetry group is *cyclic*, C_n , in which case they have exactly n rotations as symmetries (including the identity as a trivial rotation) or their group is *dihedral*, D_n , in which case they have exactly n rotations and n reflections as symmetries. Figure 1 contains a set of examples of rosettes from both types.

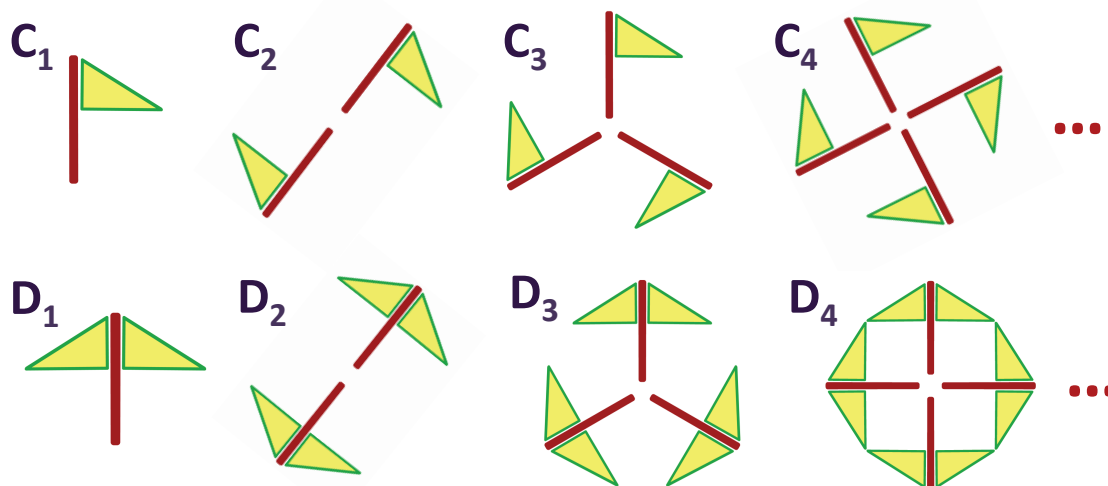


Figure 1. Examples of rosettes.

The rotational symmetries of a rosette all have the same center and are associated with rotation angles of amplitudes $360/n$ degrees (and multiples thereof). The axes of symmetry, if any, all pass through the center of rotation. In practice, to obtain n , it suffices

to identify the minimal motive which repeats itself by rotating around the center point and count the number of repetitions needed to complete the figure. If the rosette has any reflection symmetry it will be D_n , otherwise it will be C_n .

For example, a square is kept invariant if it is rotated 90 degrees clockwise around its center. This rotation is, therefore, a symmetry of the square. But there are other symmetries of the square: the rotations of 180 degrees, 270 degrees and 360 degrees, and the reflections associated with the two diagonals of the square and the two perpendicular bisectors of the pairs of parallel sides of the square. Overall the square has 8 symmetries, 4 of which are rotations (including the identity) and 4 reflections. Therefore the symmetry group of a square is D_4 .

Note that all figures with symmetry group C_1 are considered asymmetrical because they have no other symmetry, besides the identity.

A detailed description of this topic can be found, for instance, in Martin (1982) or Teixeira, Costa and Moniz (2015). Washburn and Crowe (1988) made a pioneer scholar work on the study of symmetry and its systematization. The authors not only provided a simple way to classify wallpaper patterns regarding their symmetry characteristics through an easy to follow flowchart, but they also studied patterns in various African contexts and in the American Southwest, bringing cultural relevance to their book.

The study of the relationship between mathematics and culture in mathematics education is currently known as *ethnomathematics* (D'Ambrosio, 1985, 1999) and several authors have made important contributions to this area. Ethnomathematics may be described as the study of mathematical ideas as embedded in a cultural context. It refers to the “mathematics which is practiced among cultural groups, such as national-tribal

societies, labor groups, children of a certain age bracket, professional classes, and so on”

(D'Ambrosio, 1985, p. 45). Ubiratan D'Ambrosio and Paulus Gerdes are among the most important authors within ethnomathematics, the later having made a valuable contribution to the study of geometrical characteristics, in particular symmetry aspects, of African and American patterns such as in his books (Gerdes, 2004a, 2004b, 2008).

Discovering symmetries: from the Portuguese pavement to patchwork and ceramics

The sidewalks and squares paved in Portuguese Pavement or Portuguese mosaics are one of the most characteristic aspects of the heritage of many Portuguese towns. We step on them every day but most of the time we do not pay attention to their historical, artistic and geometrical heritage.

The origins of the Portuguese pavement dates back to the mid-nineteenth century. From 1848 to 1849, the Mar Largo project was undertaken and a wave composition mosaic was built in the Dom Pedro IV Square (known as Rossio today), in Lisbon. This was an innovative project for the time. Six years before, by initiative of the same author of Mar Largo, Lt. Gen. Eusébio Pinheiro Furtado, the narrow and steep roads that access Saint Jorge's Castle had already been paved with white (limestone) and black (basalt) stones.

The Portuguese artistic pavement, characterized by the contrast between black and white colours (in general), quickly spread to the whole country, the Azores and Madeira islands being no exception. In fact, maybe due to its insular characteristic, the Azores Islands preserve a priceless heritage of Portuguese pavement which can no longer be found in other regions of Portugal.

The Portuguese pavement has also crossed international borders, naturally due to the Portuguese expansion throughout the world in the 15th and 16th centuries. Beautiful examples of Portuguese pavement can still be found in different countries. From the

famous boardwalk of Copacabana Beach in Brazil, to the Senado Square in Macau, going through countries such as Cape Verde, Angola, Mozambique, and South Africa, the Portuguese pavement has a constant presence. Still today, Portuguese pavior masters are asked to implement and teach their art abroad.

In 2013, the International Year of Mathematics of Planet Earth was celebrated. Many initiatives took place all over the world and were intended to draw attention to the central role that mathematics can play in key issues related to the Earth and to its constant presence in everyday life.

In Portugal, all the Portuguese pavement designs found across the Azores islands have been collected and classified (from a mathematical point of view) within a project that has been carried out. Symmetry itineraries have been developed that contemplated all 9 islands (see <http://sites.uac.pt/rteixeira/simetrias>). Figures 2, 3 and 4 illustrate some itineraries.

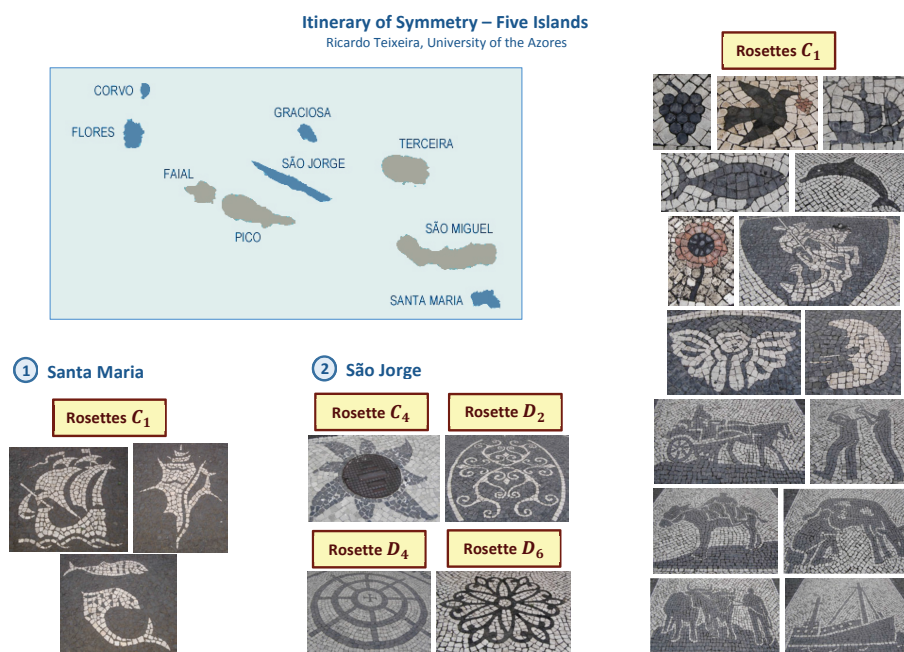


Figure 2. Detail from the Five Islands itinerary.

Rosette itinerary of Terceira island
Raquel Pereira Nogueira, Ricardo Cunha Teixeira (University of the Azores)

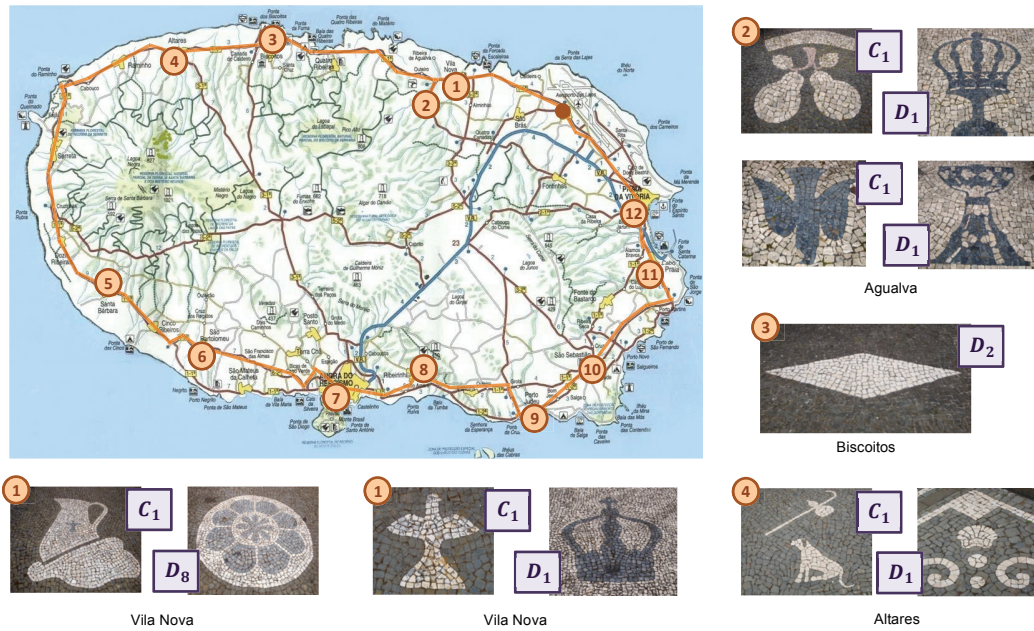


Figure 3. Detail from the rosette itinerary of Terceira island.

Itinerary of Symmetry – Pico Island
Ricardo Teixeira, University of the Azores

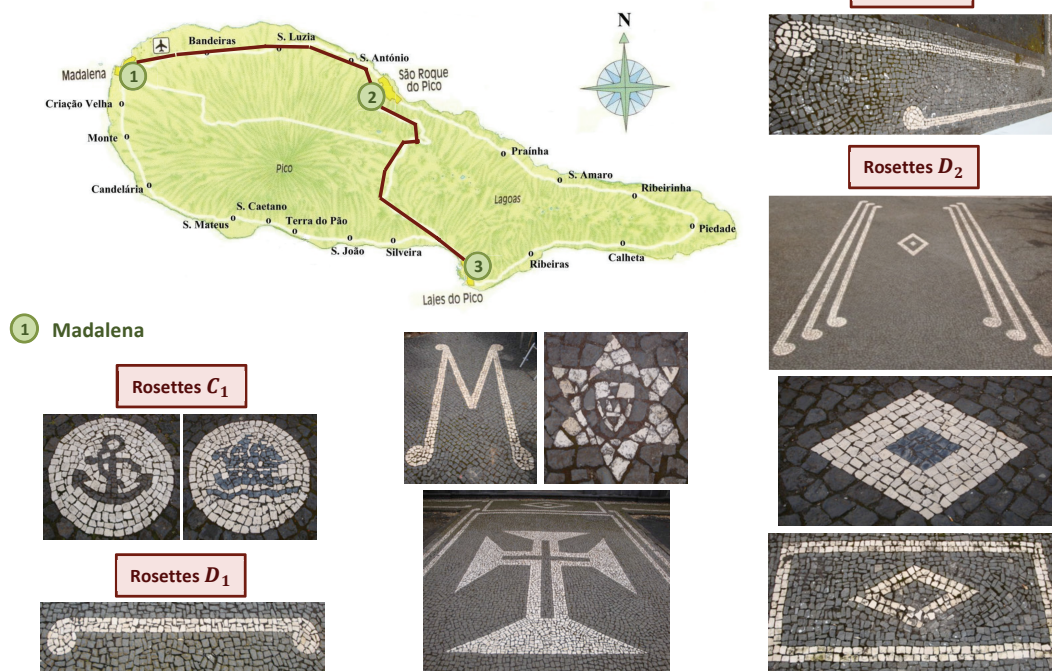


Figure 4. Detail from the symmetry itinerary of Pico island.

More recently, Carvalho *et al.* (2016) studied the symmetry properties of Lisbon sidewalks having found examples of cyclic and dihedral rosettes, all 7 types of friezes and 12 out of the 17 types of wallpaper patterns.

Besides the Portuguese pavement, various types of crafts are excellent sources of inspiration for the exploration of symmetries, interlacing mathematics and our culture. By promoting the use of mathematical tools in day to day life, symmetry can also be a good way to fight students' negative attitudes towards mathematics.

Delivering these tools to students is best done by teachers. So, it is important to first reach out to mathematics teachers and provide them with new ways to deal with the teaching of mathematics and then let them reach out to their students. Teacher training can be done in pre-service or in-service courses. Professional development courses are a powerful means to keep the education system up-to date, providing teachers with opportunities to share and to learn new strategies and methods in education.

Over the past few years, several professional development courses for mathematics teachers have taken place in Aveiro (a town in the north of mainland Portugal), focusing on the applications of mathematics to other subjects, in particular the visual arts and crafts. Some of these courses focused on the topic of symmetry (and isometry) and teachers had the opportunity to create their own piece of art using patchwork and/or ceramics under the supervision of a specialist in the area.

In the rest of this section we focus on the results of one of these professional development courses. *Rosettes in every way* (“Rosáceas a torto e a direito”) was a course for mathematics teachers which took place in 2015, from April 15th to June 27th. This course consisted of 15 hours of contact between all participants and 15 hours of individual work (including work in the teacher's classrooms). It had the collaboration of a ceramist

and a quilter who coached part of the course. The course had 17 participants who taught mathematics from grades 1 to 12. Given the wide range of grades taught and the specificity of each level of teaching, different activities were proposed for teachers of different levels (elementary and secondary).

The topic covered in this course was symmetry and isometry, with the goal of exploring rosette groups of symmetry. The general structure of the course consisted of three parts.

In the first part some concepts were provided and some activities were carried out, aiming at the deepening of the understanding and knowledge about the topic.

In the second part of the course teachers were asked to explore activities in their classroom, taking into account the specificity of its individual context. A few weeks later the experiences were shared and discussed with all the participants. Those teachers who could not work with their students (either because they didn't have any at the moment or because the timing was not appropriate) were asked to do some individual or group work related to what could be done in the classroom.

In the third part of the course teachers were asked to develop individual projects applying the concepts of the course to practical works. Each participant was challenged to create his own pieces of art. The materials used in the individual projects were fabrics and ceramics. Teachers were asked to produce examples of rosettes belonging to different groups of symmetry, from both types, C_n and D_n , with n ranging from 1 to 8. Teachers were also challenged to use, as inspiration, motives/designs from traditional Portuguese pavement.

The following figures contain photos of some of the works produced. Figure 5 contains examples of patchwork pieces with a cyclic group of symmetry. Figure 6 contains

examples of patchwork pieces with a dihedral group of symmetry. Figure 7 contains examples of ceramic pieces with a cyclic group of symmetry. Finally, Figure 8 contains examples of ceramic pieces with a dihedral group of symmetry.

Colouring is an important aspect of any art/craft work. Colouring may or may not affect the symmetry properties of a figure. In patchwork, fabrics most often contain patterns which taken rigorously destroy all the symmetries in the overall figure.

In what follows, each fabric will be considered as one-coloured. So, for instance, if a quilt uses three different fabrics, we consider it as having three colours. The classification of the symmetry group of each piece will take into account its colours, unless otherwise stated.



Figure 5. Patchwork cyclic rosettes. From left to right, top to bottom, the authors and the symmetry groups are: Amélia Sales – C_2 , Cecília Figueira – C_3 , Paula Santiago – C_4 , Ana Paula Moreira – C_4 , Dulce Mesquita – C_4 , Dora Alfaiate – C_4 .

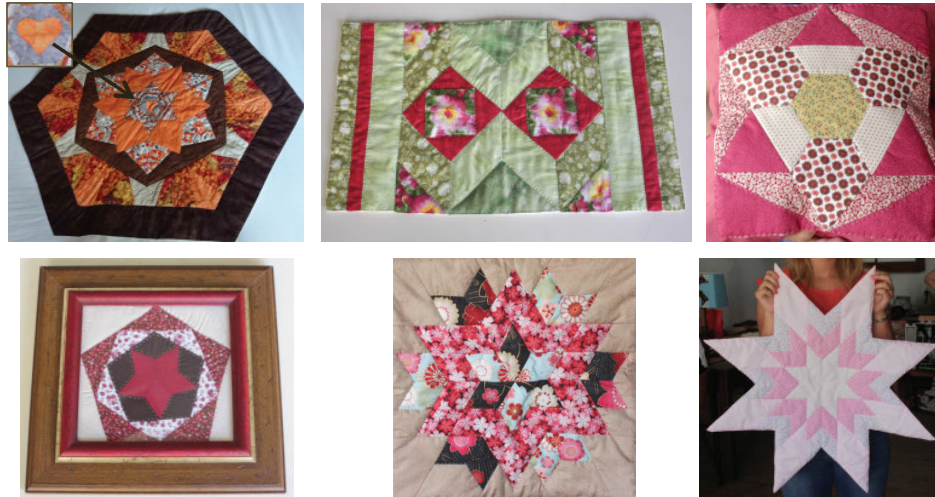


Figure 6. Patchwork dihedral rosettes. From left to right, top to bottom, the authors and the symmetry groups are: Odete Silva – D_1/ D_6 (depending on the center motive being considered or not), Maria Armanda Diz – D_2 , Ana Cristina Martins – D_3 , Susana Borges – D_5 , Andreia Ferreira – D_6 (without colouring), Joana Lemos – D_8 .



Figure 7. Ceramic cyclic rosettes. From left to right, top to bottom, the authors and the symmetry groups are: Ana Cristina Martins – C_1 , Joana Lemos – C_2 , Dora Alfaiate – C_4 , Cecília Figueira – C_5 , Amélia Sales – C_6 , Luísa Silva – C_7 .



Figure 8. Ceramic dihedral rosettes. From left to right, top to bottom, the authors and the symmetry groups are: Andreia Hall – D_1 , Ana Paula Moreira – D_2 , Susana Borges – D_3 , Dora Alfaiate – D_5 , Lucinda Belo – D_6 , Paula Santiago – D_7 .

Depending on the context, some symmetry groups are more frequent than others. Overall, even numbered symmetry groups are easier to find especially in human productions. In patchwork, several factors contribute to the preference for D_4 and C_4 (and also D_8 and C_8) designs. Right angles are the favorite because not only the threads within the fabrics cross each other's at angles of 90 degrees but also because fabric is usually bought in rectangular pieces which is easy to cut into smaller rectangular or square pieces. In addition, the need to sew together different pieces from the wrong side makes straight lines much easier to follow and produce good results.

Exploring Portuguese Pavement Rosettes with Patchwork and Ceramics

Portuguese pavements display a wide variety of designs, many of which have been specifically created for that purpose. These designs can be used as a source of inspiration for other forms of crafts and art. Originality doesn't necessarily mean creating something completely new, but rather using pre-existing ideas in new contexts.

In this section we present several patchwork and ceramic pieces inspired on pavement designs. These designs reveal a wide variety of symmetry types. Sidewalks along roads most often exhibit frieze patterns whereas public squares may present wallpaper patterns. Rosettes can be found almost everywhere with varying sizes and symmetry properties. Figure 9 contains 3 photos of beautiful large rosettes taken in the city of Ponta Delgada in S. Miguel island. Based on these, the patchwork pieces of Figure 10 were made.



Figure 9. Three pavement rosettes in Ponta Delgada, S. Miguel island.



Figure 10. Patchwork designs based on pavement rosettes, by Andreia Hall. The symmetry groups are, from left to right, C_8 , D_8 , D_8 .

Figure 11 contains a photo of a patchwork piece inspired by an amazing square pavement design found in Praia da Vitória, Terceira island.



Figure 11. Patchwork design by Paula Madeira. The symmetry group is D_4 .

Figure 12 contains a pair of pot holders, inspired by another Portuguese pavement design found in Vila da Povoação, S. Miguel. Pot holders made from crochet or fabrics are very common in Portuguese households. Traditionally they are either square or circular.

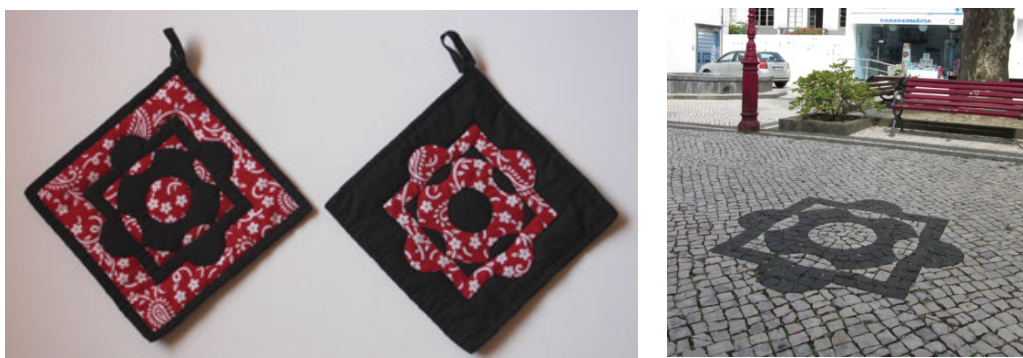


Figure 12. Patchwork pot holders by Andreia Hall. The symmetry group is D_4 .

Finally, Figures 13 and 14 refer to a set of dream catchers inspired by Portuguese sidewalk pavement designs found in Aveiro and in the Azores islands. It is curious to note that some designs can be found both in the islands and in the continent. For instance the star in the middle photo of Figure 13 taken in Aveiro, north of Portugal, can also be found in Vila da Povoação in S. Miguel island.



Figure 13. Four sidewalk Portuguese pavements. Left to right: Rua José Azevedo, Faial island, Azores; a wind rose in Corvo island, Azores; Av. Dr. Lourenço Peixinho and Rua Dr. Alberto Souto, Aveiro.



Figure 14. Ceramic dream catchers by Andreia Hall, inspired by sidewalk Portuguese pavement designs. Symmetry groups: D_4 and C_4 .

Conclusions

Interlacing mathematics with other fields of endeavor, such as the visual arts and crafts, may serve as a means of promoting the general interest in mathematics and also to improve the teaching/learning process of mathematics in the classroom. Portuguese pavements constitute a rich historical, artistic and geometrical heritage that must be valued and nourished by the present and future generations. By linking mathematics (through the study of symmetry) to the Portuguese pavement designs this paper proposes two ways to contribute to the promotion of mathematics and the preservation of the Portuguese pavements heritage.

One way is to create symmetry itineraries that can be used by both tourists and residents. These itineraries have a double effect: for those interested in mathematics, they lead them to the cultural and artistic value of traditional Portuguese sidewalks; for those interested in culture, history and travel, they draw their attention to the presence and importance of mathematics.

The other way to promote mathematics and, at the same time, value and preserve the Portuguese pavements heritage is to use these as a source of inspiration in other crafts, not only by craftsmen but also by teachers and students in mathematics classes. Within the general topic of geometry, the study of isometries and symmetries can be facilitated by applied tasks. Analyzing the symmetry properties of craft works, including sidewalk designs is a good way to help learning these topics. One further step is to create our own projects, involving symmetry and eventually inspired by pre-existing materials. A well-known citation by Confucius says: “I hear and I forget, I see and I remember, I do and I understand”. Creating our own projects (making the designs, choosing the materials and doing the work) is a very appealing way to learn and indeed understand the topics

addressed. The professional development course described in this paper has showed us that by creating their own projects, teachers reinforced their knowledge about symmetry and became familiar with rosette groups of symmetry. They revealed a good understanding of the topic and above all they seemed very pleased and contented while doing their projects – they were having fun while learning. In addition, by using traditional designs to inspire their creations, teachers became more aware of their cultural heritage. This is a similar approach to the way D'Ambrosio (1985, 1999) presents Ethnomathematics, claiming that Ethnomathematics is the study of mathematical ideas as embedded in a cultural context. Teachers told us that they now walk the streets with a different attitude: they look at pavements and tiles which went unnoticed in the past and intuitively they tend to analyze the symmetry properties of the designs they see. Having this awareness, teachers might also foster in their students a change in the way they see mathematics in everyday life. As a concluding remark, we believe that it is very rewarding to promote activities (such as the one described in this paper) that make mathematics more meaningful, both to the general public and in the mathematics classroom. The projects addressed in this paper show that mathematics is present in many real life situations and that culture, mathematics, arts/crafts and education do not stand alone and can be worked together.

Acknowledgments

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